Iterative Axisymmetric Identification Algorithm (IAIA) for real-time reconstruction of the plasma boundary of ITER

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Abstract

A new boundary reconstruction procedure is presented and validated against ITER nominal equilibria. An approximation of the plasma with an equivalent filamentary current model is employed, which is computed iteratively and allows to describe a wide variety of plasma current distributions (from the peaked ones, to the pedestal current ones). One of the specific features of the procedure is how the filaments are switched on and how the total current is distributed over the entire set, being the filaments independently considered: this allows more degrees of freedom to the model to adapt to particular current distributions, yielding better performances with a negligible additional computational burden. The code also implements a special points search making it well suited for both diverted (be they top or bottom x-point) and limiter configurations. In addition also the reconstruction in presence of noise has been explored.

Keywords: boundary reconstruction, equivalent currents, real-time, ITER

1. Introduction

Over the years, the research on efficient techniques to guarantee an accurate and real-time reconstruction of the plasma boundary from magnetic signals has been a central issue, since such procedures are of paramount importance for both the safe operation of the fusion device and for reaching the expected performance of the designed configuration by means of plasma position and shape feedback systems.

As an alternative to full MHD equilibrium reconstruction codes [1, 2, 3], other methods, mainly based on semi-analitycal procedure to approximate the plasma flux distribution such as *Equivalent Currents* [4, 12, 6], *Toroidal Harmonics* [5] or *Local Field Expansion* [7, 8], have been proposed to comply with the specific requirements of real-time plasma contour identification. On the ground of the results from the studies carried out for ITER and other tokamaks [9], a new boundary reconstruction procedure has been developed, based on an approximation of the plasma with a set of equivalent currents, adaptively allocated within the plasma domain, to obtain the best estimate of the plasma contour.

In this work, the proposed procedure is validated against some ITER reference equilibrium configurations. In addition to the reconstruction of the nominal equilibria also the reconstruction in presence of noise has been explored.

2. Description of the machine

The ITER magnetic diagnostics [14] comprise several subsystems and different kind of sensors (pick-up coils, saddle coils, flux loops, Rogowski coils). This study is focused on the sub-set of equilibrium sensors (pick-up coils) mounted on the inner wall of the vacuum vessel (24 tangential and 12 normal coils) and on the divertor diagnostic cassettes (12 sensors), shown in Fig. 1-left. The criteria to assess the boundary reconstruction capability of a sensor set refer to local and to global concepts [11]. According to the local approach, in this study two sets of boundary to first wall distances (gaps) are used as the descriptors of the plasma shape:

- a set of 6 control gaps intended for shape control and named *control gaplines*;
- a set of 24 gap points (defined as points along the first wall), intended as check points for the limiter condition and named *diagnostic points*.

While for the former set of gaps, the distance from the first wall along the gapline is taken as the *gap value*, for the latter a procedure has been implemented, that mimics the control gaplines and considers the distance along a line orthogonal to the first wall, starting at the diagnostic point and measuring $1m$ inwards, thus defining a set of *diagnostic gaplines*, (Fig. 1-right).

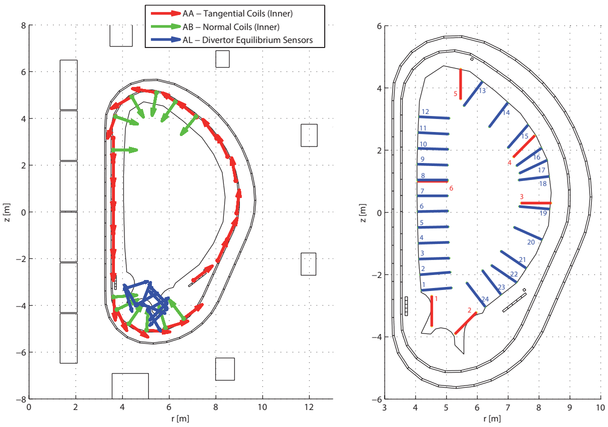

Figure 1: ITER cross section. Left: inner pick-up coils (AA, AB, AL). Right: 6 control (red) and 24 diagnostic (blue) gaplines.

3. Equivalent current model

The identification of the plasma magnetic contour γ_p[1] can be cast as an inverse problem where a set of Equivalent Currents (ECs) have to be determined in order to approximate the plasma contribution to the magnetic measurements. The number and the positions of the ECs are degrees of freedom of the inverse identification problem and they must be *a priori* assigned [12, 13].

3.1. Solution of the inverse problem

When the currents induced in the conducting structures surrounding the plasma are negligible (e.g. in *steady state*), the n_m magnetic measurements $\mathbf{m}$ can be expressed as a linear combination of the effects from the n_c active coil currents $\mathbf{i}_c$ and the n_p ECs $\mathbf{i}_p$

$$\mathbf{G}_c\,\mathbf{i}_c + \mathbf{G}_p\,\mathbf{i}_p = \mathbf{m}, \tag{1}$$

where the elements of the Green matrices $\mathbf{G}_c$, $\mathbf{G}_p$ are computed via numerical integration using standard closed formulas. Then, $\mathbf{i}_p$ can be calculated from

$$\min_{\mathbf{i}_p} \left\|(\Sigma\mathbf{G}_p)\mathbf{i}_p - \Sigma\mathbf{m}_p\right\|_2 \tag{2}$$

where $\mathbf{m}_p = \mathbf{m} - \mathbf{G}_c\mathbf{i}_c$ represents the measurements $\mathbf{m}$ from which the known effect of the external coil currents $\mathbf{i}_c$ is subtracted, $\|\cdot\|_2$ is the Euclidean norm, and the pre-multiplication of the residual vector by the diagonal matrix $\Sigma = diag(\sigma_i)$ allows the consideration of different levels of measurement noise (e.g. standard deviations σ_i). In practice, the numerical solution of (2) is ill-conditioned ($n_m \neq n_p$) and a regularization method is needed to obtain a unique regular solution approximating the desired vector $\mathbf{m}_p$. The extent of the ill-conditioning is strongly dependent both on the probe locations (a crucial issue in the design of new machines) and on the arrangement selected for the equivalent currents (see 3.2). A Singular Values Decomposition (SVD) technique is adopted to approximate the ill-conditioned matrix $\mathbf{G}_p$ with a better-conditioned one. An automated procedure can be implemented to provide the correct SVD truncation index k and guarantee that the residual of (2) is within the measurement errors [13].

Interestingly, the effect of a measurement perturbation $\delta\mathbf{m}_p$ propagates to the solution of (2) according to

$$\frac{\|\delta\mathbf{i}_p\|}{\|\mathbf{i}_p\|} \le c(\mathbf{G}_p)\frac{\|\delta\mathbf{m}_p\|}{\|\mathbf{G}_p\|\,\|\mathbf{i}_p\|} \tag{3}$$

where the conditioning number $c(\mathbf{G}_p)$ of the system matrix $\mathbf{G}_p$ acts as an amplification factor with respect to the perturbations on the measurement errors $\delta\mathbf{m}_p$.

3.2. Iterative procedure

In order to place the ECs on the poloidal cross section within the plasma domain, a set of n_r rays is defined (each ray starts at the machine centre and extends towards the first wall). Along these rays, the ECs are *switched on/off* according to the following procedure:

1. A first set of n_1 ECs is placed at the beginning of the current rays, in an area that is basically included in any plasma cross section shape, and an initial guess of the plasma boundary is computed;
2. A second set of n_2 ECs is generated along the rays, midway between the starting point of the ray and the currently identified boundary[2], and a further boundary is computed; the current distribution among the filaments is obtained according to (2);
3. Step 2 is iterated until convergence.

Fig. 2 shows an example of the iterative procedure: the light blue lines are the n_r rays, the black dots denote the locations of the equivalent currents *switched on*, the red line is the reference boundary and the blue dots are the discrete version of the reconstructed boundary[3].

[1] γ_p is defined as the outermost closed flux line touching the first wall in one point (*limiter* configuration) or passing through a point of null magnetic field (*diverted* configuration).

[2] To avoid local artifacts induced by the filamentary model, the ECs are anyway kept at a minimum distance from the identified boundary.

[3] The discrete version of the boundary is computed as a set of gap distances along the current rays: the actual reconstructed boundary is then produced by smoothing of this gap sets, thus ensuring more robustness and regularization to the solution.

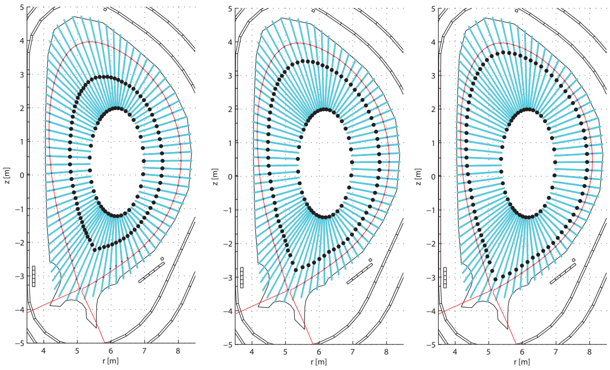

Figure 2: ECs location. From left to right: three successive iteration steps moving the second array (n_2 ECs) towards the plasma boundary.

4. Results

The IAIA code has been validated against a wide equilibrium data set of some hundred magnetic configurations characterized by different values of the main plasma parameters: toroidal current I_p, normalized internal inductance ℓ_i, poloidal beta β_p, elongation κ, lower and upper triangularity δ_{low}–δ_{up}. Three equilibria (Tab. 1) have been selected for the specific analysis described in the next subsections; their reference flux map and plasma current density are shown in Fig.3.

Table 1: Main plasma parameters of the sample equilibria.

	I_p	ℓ_i	β_p	κ	δ_{low}	δ_{up}
#39 (*x-point*)	15.0	0.59	0.47	1.94	0.54	0.43
#131 (*limiter*)	2.53	1.87	0.19	1.30	0.10	0.08
#230 (*x-point*)	9.36	0.75	1.97	1.81	0.54	0.35

The results of the reconstruction procedure are summarized in Fig.4. On the left, the final locations of the equivalent current sets (n_1, n_2); on the middle, the gap values reconstructed by the IAIA code (red dots) are superimposed to the reference boundary (red line); on the right, from top to bottom, the absolute errors of the reconstructed control and diagnostic gaps, respectively.

To assess the boundary reconstruction performance in presence of noise[4], for each sensor the *measured* signal y_m is calculated from its *ideal* value y according to

$$y_m = (1 + \epsilon_r)\, y + \epsilon_a\, y_0 \tag{4}$$

where ϵ_r (normal distribution with zero mean, standard deviation σ_r) takes into account the errors proportional to the actual value of the signal, whilst ϵ_a (normal distribution with zero mean, standard deviation σ_a) takes into account the errors independent on the actual value, normalized to a typical field value $y_0 = 2.1\,T$. As a test case, two levels of noise have been applied to the input data (*ideal* signals) of the reference configuration #39:

- Noise level 1 (NL1): $\sigma_r = 0.7\%, \sigma_a = 0$
- Noise level 2 (NL2): $\sigma_r = 0.35\%, \sigma_a = 0.2\%$

For each level, the reconstruction procedure described in Sec.3 has been executed 100 times in order to produce reliable statistics (mean values, standard deviations) of the gap errors; the results are summarized in Fig.5.

[4] Magnetic diagnostics are usually affected by several error sources, such as the non ideality of the measurement chain or parasitic effects (eddy currents, ferromagnetic inserts, radiation).

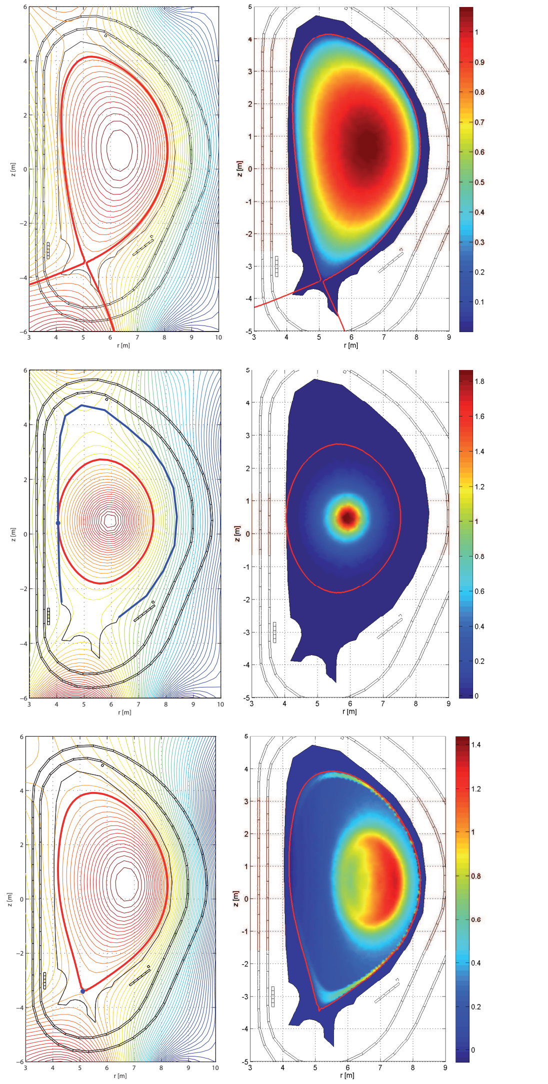

Figure 3: From top to bottom: equilibria #39, #131, #230. Reference flux map and plasma current density.

Conclusions

A boundary reconstruction procedure based on a novel concept of plasma equivalent filamentary model

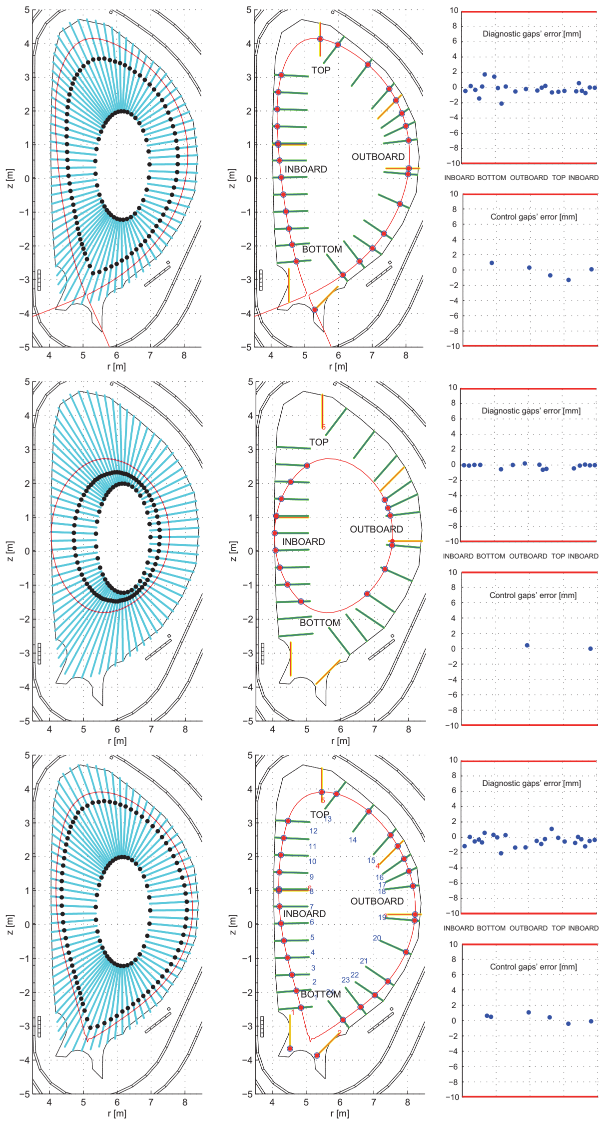

Figure 4: From top to bottom: equilibria #39, #131, #230. From left to right: final locations of the equivalent currents, gaps reconstruction and corresponding absolute errors.

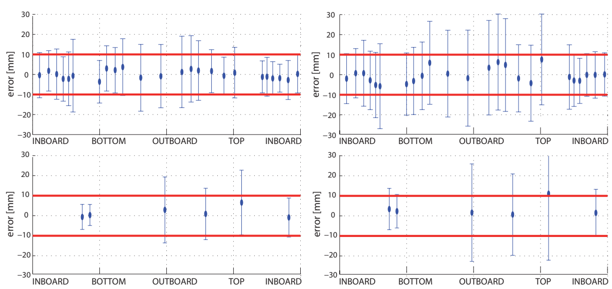

Figure 5: Boundary reconstruction for sample equilibrium #39. Noise analysis. Left: NL1, Right: NL2.

has been developed and validated against ITER nominal equilibria. One characteristic feature is how the filaments are *switched on* and how the total current is distributed over the entire set, with the filaments being independently considered to allow more degrees of freedom to the model. Actually, this approach brings in the capability of fitting specific current distributions, yielding to better performances in the boundary reconstruction with a negligible additional computational burden. This is particularly remarkable in the case of equilibria characterized by plasma pedestal currents as well as small plasmas. The code also implements a special point search making it well suited for diverted (be they top or bottom x-point) as well as limiter configurations.

The performance of the reconstruction code is satisfactory in all the configurations under test with reconstruction errors of few millimeters in the nominal cases, errors that reasonably grow in presence of noise (especially when the average distance between the plasma boundary and the first wall is large).

Acknowledgments

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